

Section 8.3: Systems of Linear Equations – Determinants

Steps to find the determinant of a 2 by 2 matrix.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
$$\det(A) = ad - bc$$

For example: Find the determinant of A:

$$A = \begin{vmatrix} 5 & 6 \\ 2 & 4 \end{vmatrix} = 5(4) - 2(6) = 20 - 12 = 8$$

Steps to find the determinant of a 3 by 3 matrix

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a \cdot \det \begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \cdot \det \begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \cdot \det \begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

For Example: Find the determinant of $\begin{bmatrix} 1 & 3 & 2 \\ -3 & -1 & -3 \\ 2 & 3 & 1 \end{bmatrix}$

$$\begin{aligned} \det \begin{bmatrix} 1 & 3 & 2 \\ -3 & -1 & -3 \\ 2 & 3 & 1 \end{bmatrix} &= 1 \cdot \det \begin{bmatrix} -1 & -3 \\ 3 & 1 \end{bmatrix} - (3) \cdot \det \begin{bmatrix} -3 & -3 \\ 2 & 1 \end{bmatrix} + 2 \cdot \det \begin{bmatrix} -3 & -1 \\ 2 & 3 \end{bmatrix} \\ &= 1[-1 - (-9)] - 3[-3 - (-6)] + 2[-9 - (-2)] \\ &= 1(-1 + 9) - 3(-3 + 6) + 2(-9 + 2) \\ &= 1(8) - 3(3) + 2(-7) \\ &= 8 - 9 - 14 \\ &= -15 \checkmark \end{aligned}$$

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Cramer's Rule (2x2)



Let A be the coefficient matrix for the system:

$$ax + by = e$$

$$cx + dy = f$$

If $\det A \neq 0$, then the system has one solution, and

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A} \quad \text{and} \quad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}$$

Here is how to use Cramer's rule (minus the misspelling of his name)

Cramer's Rule 2x2

$$\begin{array}{l} x - y = 4 \\ 2x + y = 2 \end{array} \quad \begin{array}{l} x = ? \\ y = ? \end{array} \quad x = \frac{D_x}{D} \quad y = \frac{D_y}{D}$$

$$D = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 3$$

$$x = \frac{D_x}{D} = \frac{6}{3} = 2$$

$$D_x = \begin{vmatrix} 4 & -1 \\ 2 & 1 \end{vmatrix} = 6$$

$$y = \frac{D_y}{D} = \frac{-6}{3} = -2$$

$$D_y = \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} = -6$$

#1 – 8: Find the value of the determinant of the following matrices

$$1) \begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix} \quad 2) \begin{bmatrix} -2 & 4 \\ 3 & 1 \end{bmatrix} \quad 3) \begin{bmatrix} 3 & -5 \\ 0 & -2 \end{bmatrix} \quad 4) \begin{bmatrix} -4 & 5 \\ -1 & 0 \end{bmatrix}$$

$$5) \begin{bmatrix} 3 & 5 & -1 \\ 4 & 2 & 3 \\ 2 & 5 & 6 \end{bmatrix} \quad 6) \begin{bmatrix} 1 & 0 & -1 \\ 7 & 2 & 4 \\ 2 & 3 & -5 \end{bmatrix} \quad 7) \begin{bmatrix} -3 & 2 & 8 \\ 0 & 6 & -3 \\ 2 & 1 & 4 \end{bmatrix} \quad 8) \begin{bmatrix} 3 & 2 & 0 \\ 4 & -1 & 3 \\ 1 & -5 & 6 \end{bmatrix}$$

Solve the system of equations using Cramer's rule

$$9) \begin{cases} 6x + 2y = 10 \\ 2x - y = 5 \end{cases}$$

$$10) \begin{cases} 8x - 3y = -2 \\ 2x + y = -4 \end{cases}$$

$$11) \begin{cases} 4x - 3y = -2 \\ x - 5y = -9 \end{cases}$$

12)

$$\begin{cases} 5x + 2y = 23 \\ x - 7y = -25 \end{cases}$$

$$13) \begin{cases} 5x + y = -7 \\ 3x - 2y = -12 \end{cases}$$

$$14) \begin{cases} 4x + y = 11 \\ 3x - 5y = 37 \end{cases}$$

$$15) \begin{cases} 3x + 2y = 11 \\ 2x - y = 5 \end{cases}$$

$$16) \begin{cases} 4x - 3y = 10 \\ 2x + y = 10 \end{cases}$$

$$17) \begin{cases} 4x - 2y = 7 \\ 2x - 5y = -3 \end{cases}$$

$$18) \begin{cases} 6x + 2y = 26 \\ 2x - 7y = 24 \end{cases}$$

$$19) \begin{cases} 5x + 2y = 22 \\ 3x - 5y = 7 \end{cases}$$

$$20) \begin{cases} 4x + 2y = 3 \\ 3x - 5y = 1 \end{cases}$$

21)

$$\begin{aligned} -x + y + 2z &= 1 \\ 2x + 3y + z &= -2 \\ 5x + 4y + 2z &= 4 \end{aligned}$$

22)

$$\begin{aligned} 3x - 2y + z &= 15 \\ -x + y + 2z &= -10 \\ x - y - 4z &= 14 \end{aligned}$$

23)

$$\begin{aligned} -5x - y + 3z &= -14 \\ -2x + 2y - 6z &= 16 \\ x + 7y + 2z &= -5 \end{aligned}$$

24)

$$\begin{aligned} 4x + 4y + 4z &= 12 \\ 4x - 2y - 8z &= -12 \\ 5x + 3y + 8z &= 21 \end{aligned}$$

25)

$$\begin{aligned} -x + 2y - z &= -17 \\ 2x - y + z &= 21 \\ 3x + 2y + z &= 19 \end{aligned}$$

26)

$$\begin{aligned} x + y + 2z &= 6 \\ 2x + 3y + z &= 11 \\ 5x + 4y + 2z &= 19 \end{aligned}$$

27)

$$\begin{aligned} 4x + y + z &= 9 \\ 3x - 2y + z &= 4 \\ 5x - 4y + z &= 6 \end{aligned}$$

28)

$$\begin{aligned} x - y + z &= 2 \\ 2x + y + z &= 5 \\ 7x + 4y - z &= 9 \end{aligned}$$